ΜΕΘΟΔΟΙ PREDICTIVE ANALYTICS ΣΕ BIG MOBILITY DATA ΚΑΙ ΙΑΤΡΙΚΕΣ ΕΦΑΡΜΟΓΕΣ

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> 12 Δεκ. 2018 @ Ινστιτούτο Πληροφοριακών Συστημάτων (ΙΠΣΥ), Ερευνητικό Κέντρο "Αθηνά"





Overview

- DART Data-driven AiRcraft Trajectory prediction research
- datAcron Big Data Analytics for Time-Critical Mobility Forecasting
- Track & Know R&D in Big Data Mobility in transport, insurance & health
- ★ Assurance Sparse model learning in health Informatics (fMRI) → EKΠA

- * See: H. Georgiou et al. (2018), "Predicting the next steps of moving objects: A survey". (under preparation)
- * See: Y. Kopsinis, H. Georgiou, S. Theodoridis (2014). "fMRI Unmixing Via Properly Adjusted Dictionary Learning", 22nd European Signal Processing Conference (EUSIPCO 2014), 1-5 Sept 2014 @ Lisbon, Portugal.













- The Data Science Lab at the Univ. Piraeus, established in 2015
- Aims advance research on a wide range of Data Science subjects, including:
 - big data management
 - statistics and data (incl. text, audio) analytics
 - machine learning
 - semantic integration
 - mobility data exploration
 - data privacy







Data-driven AiRcraft Trajectory prediction research

Founding Members







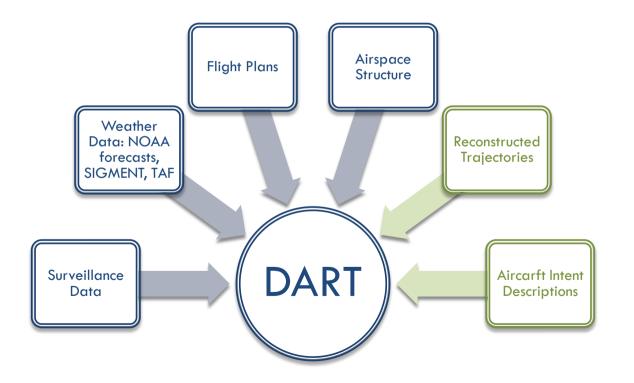


- Trajectory Prediction in the Aviation domain is devoted to the analysis and evaluation of a wide range of data-driven techniques that could potentially be applied to the aircraft trajectory prediction problem.
- Main specific research objectives addressed:
 - Study of the application of *Big-data* techniques to trajectory-related data gathering, filtering, storing, prioritization, indexing and segmentation to support the generation of reliable and homogenous/fused input datasets.
 - Study of different data-driven learning techniques to describe how a reliable trajectory prediction model will leverage them.
 - Exploration of advanced visualization processes for data-driven model algorithms design, tuning and validation, in the context of 4-D trajectories.

^{*} See: H. Georgiou et al. (2018), "Predicting the next steps of moving objects: A survey". (under preparation)

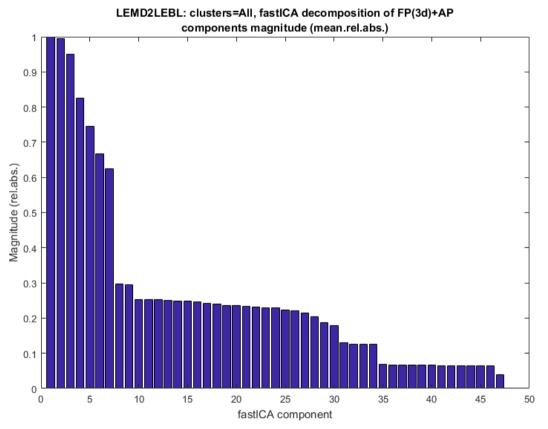






Example: Madrid/Barcelona route (about 700 flights per month per direction)

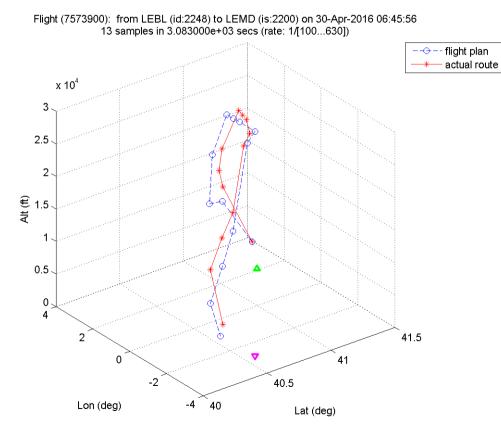




TP-C: fastICA spectrum of FP/RT dataset, Lat-only



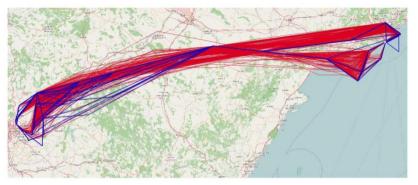




Basic idea – Method outline:

- 1. <u>Input</u>: Flight plans, actual routes, local weather, aircraft type, (...)
- 2. Stage-1: cluster the actual routes, produce semantic-aware medoids as "representatives"
- **3. Stage-2**: Build a **Pred.Model** for each medoid, associate it with the cluster flight plans (emissions)
- 4. Stage-3: For a new flight plan, find the k closest matches (Pred.Model)
- 5. <u>Output</u>: $k \ge 1$ best-estim. (HMM: true, LR: synthetic) of the query FP, for prob.estim. or further refinement





Input: flight plans (blue) and actual routes (red)

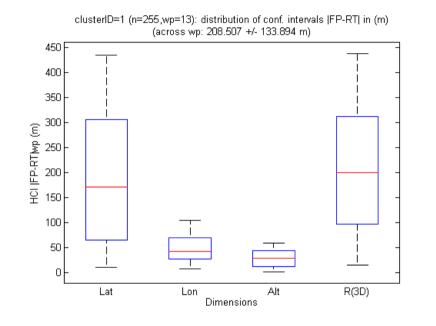


Output: semantic-aware cluster medoids (colored)

Step 1: clustering semantically annotated trajectories

- <u>Input</u>: Reference waypoints, matched with flight plans
- Each waypoint is enriched with semantics, i.e., local weather, aircraft type, etc.
- Use semantic-aware similarity metric, not just spatio-temporal track data
- <u>Output</u>: Semantic-aware cluster medoids





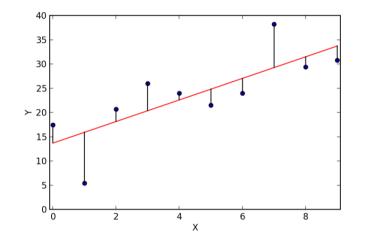
Step 2: building HMM for prediction

- <u>Input</u>: Enriched flight plans + medoid for each cluster, query FP
- <u>Output</u>: Max. likelihood estim. (emissions) of per-waypoint FP/RT deviations for query FP
- HMM approach enabled us to quickly estimate the confidence intervals (HWCI) of the accuracy, s.t. further experiments.

Route $(q)_j \approx \text{FlightPlan}(q)_j - \text{MeanDeviation}(k)_j \pm \text{HWCI}(k)_j$

* See: H. Georgiou et al. (2018), Semantic-aware aircraft trajectory prediction using flight plans. (submitted)





Step 2: building Linear Regr. for prediction

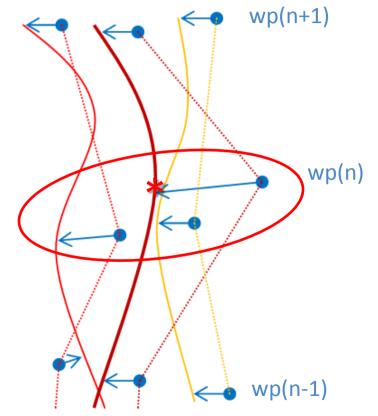
- <u>Input</u>: Enriched flight plans for each cluster, query FP
- <u>Output</u>: LSE-optimized estim. of perwaypoint value of predicted route
- The previous HMM approach is functionally a special case of LR, which is more generic.

Route $(q)_j \approx \text{FlightPlan}(q)_j * B_j + BO_j \pm HWCI(k)'_j$

(DT: per-node/leaf model)

Note(1): Decision Tree (DT) regressor can be used instead or LR as replacement (e.g. CART). Note(2): HWCI for LR error here is expected to be at most equal to the corresponding HWCI for HMM.



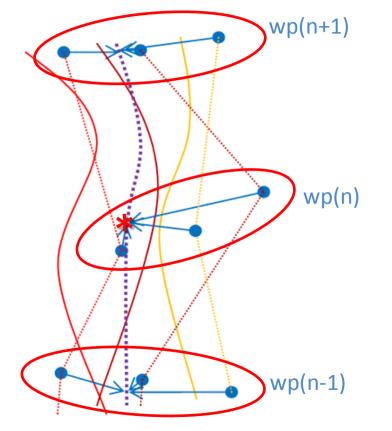


Stage-2, HMM approach:

RT(n)-FP(n) statistics are used to build a probabilistic model (HMM emissions) for ref. point wp(n).

dotted line: flight plan (FP), solid line: actual route (RT) arrows: FP/RT deviations, star: current pred. point (wp(n)) bold solid line: cluster medoid, pred. route for query FP





Stage-2, LR or DT approach:

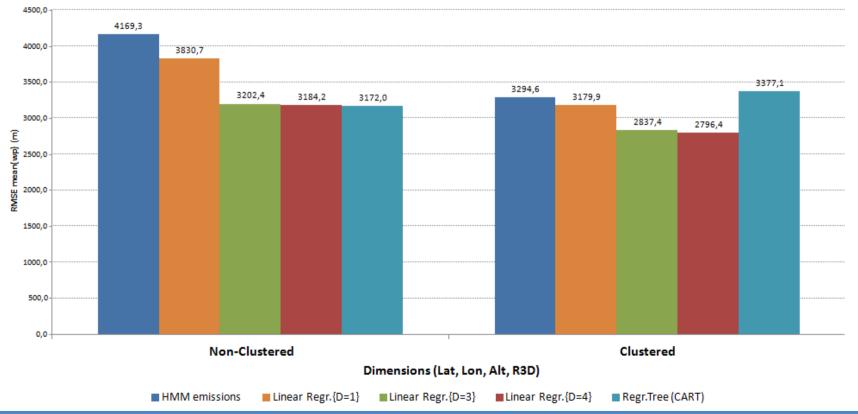
RT(n) is estimated as synthetic from multiple/all FP(*) ref. points, used to build a LSE-minimum linear prediction model.

dotted line: flight plan (FP), solid line: actual route (RT) arrows: FP/RT deviations, star: current pred. point (wp(n)) bold solid line: cluster medoid, pred. route for query FP

Overview: TP performance tests



Dataset: Madrid/Barcelona, April 2016 (LEMD2LEBL)



Non-linear Regressors (NN)

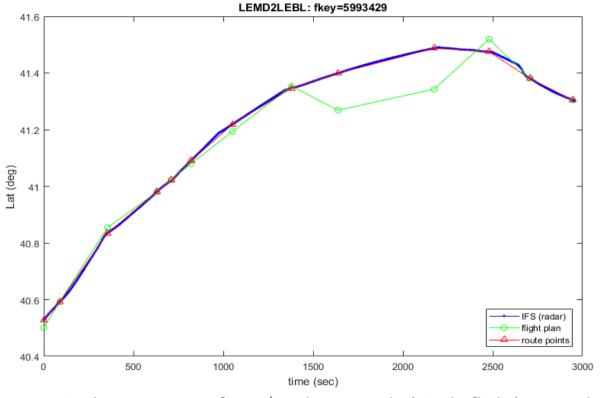


		NN/MLP/h10(BR): RMSE (m)			
Nk	Output:	RT(j)Lat	RT(j)Lon	RT(j)Alt	RT(j)R3D
696	Input:				
Nwp	wp1	240,7	139,3	31,5	279,9
11	wp2	105,0	71,1	67,1	143,5
	wp3	1500,4	301,2	221,6	1546,3
	wp4	2813,7	1424,6	244,0	3163,2
	wp5	1709,7	1298,6	342,8	2174,2
	wp6	1519,2	670,9	235,9	1677,4
	wp7	1314,6	1744,8	252,3	2199,1
	wp8	2255,7	1808,6	172,7	2896,4
	wp9	1928,0	832,7	189,5	2108,7
	wp10	2033,6	764,4	215,5	2183,2
	wp11	1691,6	1504,9	269,9	2280,1
	mean:	1555,7	960,1	203,9	1877,4
	stdev:	798,6	632,4	88,7	943,4
	696 Nwp	696 Input: Nwp wp1 11 wp2 11 wp3 wp4 wp5 wp6 wp7 wp8 wp9 wp10 wp11 wp1 wp11	Nk Output: RT(j)Lat 696 Input: Nwp wp1 240,7 11 wp2 105,0 11 wp2 105,0 wp3 1500,4 wp4 2813,7 wp5 1709,7 wp6 1519,2 wp7 1314,6 wp8 2255,7 wp9 1928,0 wp10 2033,6 wp11 1691,6 wp11 1691,6 mean 1555,7	Nk Output: RT(j)Lat RT(j)Lon 696 Input: Nwp wp1 240,7 139,3 11 wp2 105,0 71,1 wp3 1500,4 301,2 wp4 2813,7 1424,6 wp5 1709,7 1298,6 wp6 1519,2 670,9 wp7 1314,6 1744,8 wp8 2255,7 1808,6 wp9 1928,0 832,7 wp10 2033,6 764,4 wp11 1691,6 1504,9 wp11 1691,6 1504,9 mean 1555,7 960,1	Nk Output: RT(j)Lat RT(j)Lon RT(j)Alt 696 Input: Nwp wp1 240,7 139,3 31,5 11 wp2 105,0 71,1 67,1 wp3 1500,4 301,2 221,6 wp4 2813,7 1424,6 244,0 wp5 1709,7 1298,6 342,8 wp6 1519,2 670,9 235,9 wp6 1519,2 670,9 235,9 wp7 1314,6 1744,8 252,3 wp8 2255,7 1808,6 172,7 wp9 1928,0 832,7 189,5 wp10 2033,6 764,4 215,5 wp11 1691,6 1504,9 269,9 wp1 1491,6 1504,9 269,9 wp1 1691,6 1504,9 269,9 wp1 1691,6 1504,9 203,9 wp1 1691,6 1504,9 <t< td=""></t<>

LEMD2LEBL (CV.k=10): input=FP(3D)+AP \rightarrow output=Lat/Lon/Alt (NN)

Advanced pre-processing: Wiener filtering

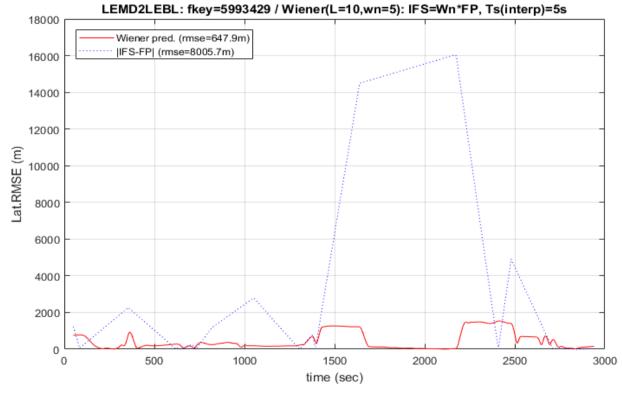




TP-C: Close-up view of a FP/RT data sample (single flight), Lat-only

Advanced pre-processing: Wiener filtering

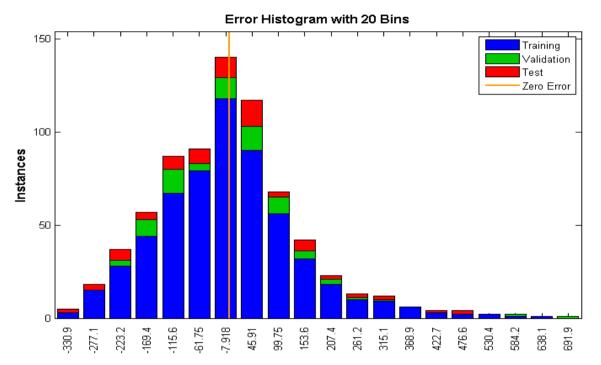




TP-C: Wiener forward predictor for FP/RT (single flight), Lat-only

Special predictive modeling: ETA (aviation)





Errors = Targets - Outputs

NN/MLP: ETA pred. errors on FP/RT dataset (top-20 of 217 stat.feat.)





datAcron

Big Data Analytics for Time-Critical Mobility Forecasting



Grant Agreement No: 687591





Objective	Current Performance	Target Performance	
Computation of data synopses	<95% compression	>95% data compression without harming the quality of analytics results	
Real-time trajectory reconstruction	Offline trajectory reconstruction	Real-time computations	
Efficient large-scale mobility data analytics	Gb size of datasets	Tb size of datasets	
Real-time trajectories forecasting for ATM and maritime, resp.	Short forecasting horizon, depending on current speed of airplane/vessel.	Increase the accuracy in positional predictions by reducing the standard deviation of the positional error in prediction.	

Task 2.2 Prediction components

- 1. Future Location Prediction (FLP):
 - **Short-term (FLP-S)**: Online, based only on recent positions (time series), lookahead time is a few minutes at most.

- **Medium/Long-term (FLP-L)**: Online, based on recent positions and history (routes network), look-ahead time is up to entire trips (end-to-end).

- 2. Trajectory Prediction (TP):
 - Long-term, Unconstrained (TP-U): "Only the starting point is specified"

 \Rightarrow Addressed via FLP-L (routes network).

- Long-term, Destination-only (TP-D): "Staring and ending points are specified"

 \Rightarrow Addressed via FLP-L (routes network).

- Long-term, Constraints-based (TP-C): "Several reference points are specified"

 \Rightarrow Offline/batch-only, exploiting complete & enriched flight plans.



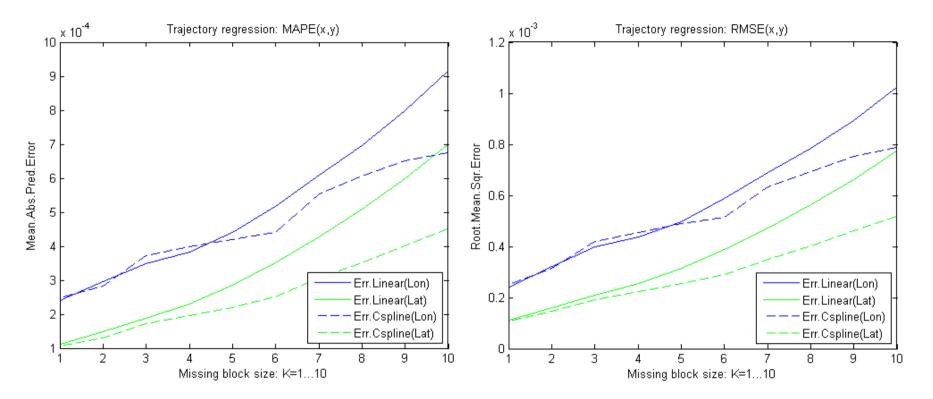


Task 2.2 Prediction components

- 1. Distributed online FLP (FLP-S): Extension of the RMF algorithm to a distributed P-RMF* for short-term FLP (*Petrou et.al. 2018*)
 - Using surveillance data + predefined motion patterns
 - Improvement: Bypass the problems of original RMF on real data
- 2. Distributed online Medium/Long-term (FLP-L)
 - Designed and at a good level of implementation
- **3.** Semantically-driven TP (TP-C): Novel hybrid clustering/LR-based method for TP (*Georgiou et al. 2018*)
 - Using enriched trajectories + prior constraints (e.g. flight plans) for use on TP-C tasks
 - Improvement: HMMs replaced by more efficient Linear Regressors



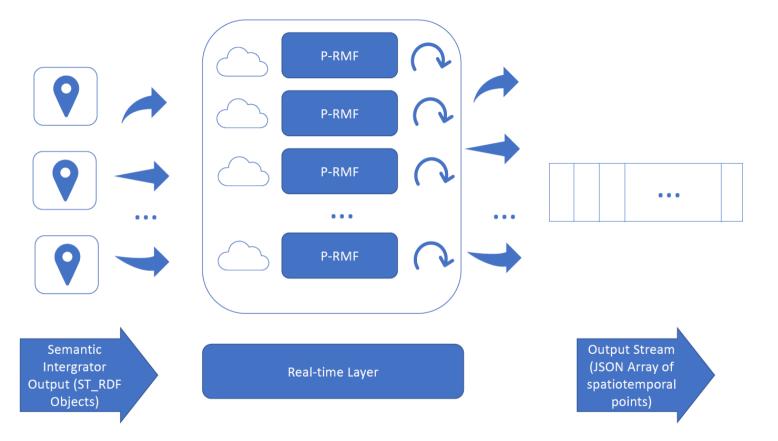




FLS-S: Close-up view of extrapolation errors, horizontal-only







FLP-S (a.k.a P-RMF*) architecture



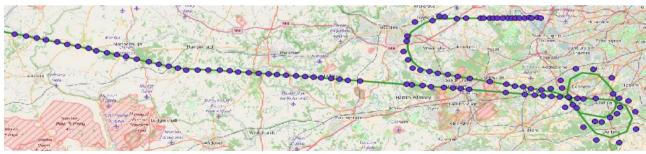


P-RMF* in Action (aviation, London landing)

Flight Aware data (London): Id= 10, Sampling rate= 8sec, Horizon= 8



a) actual flight landing

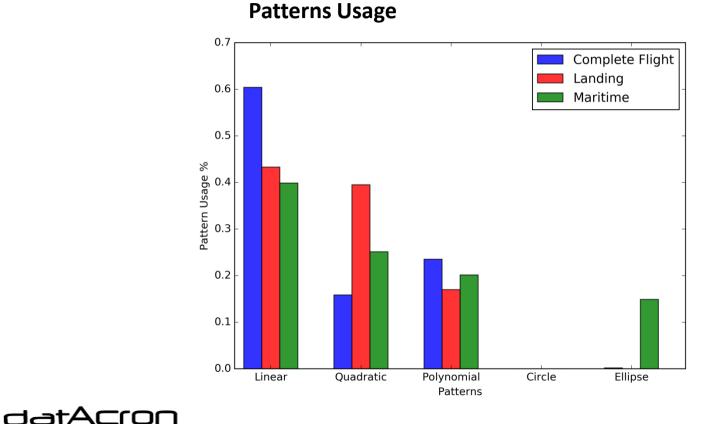


b) predicted future point (H=8)



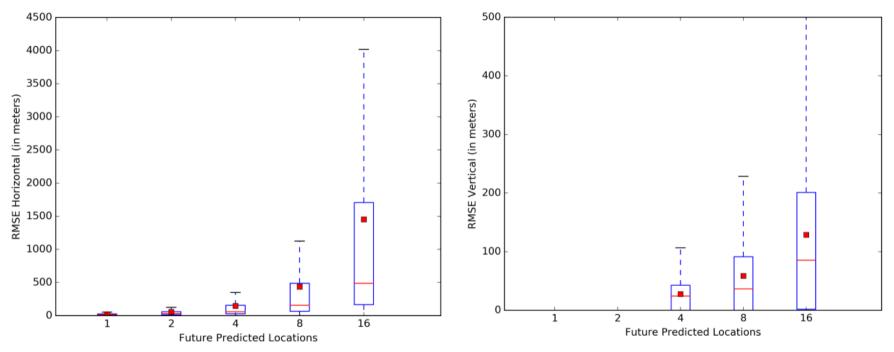


P-RMF* in Action (aviation, maritime)



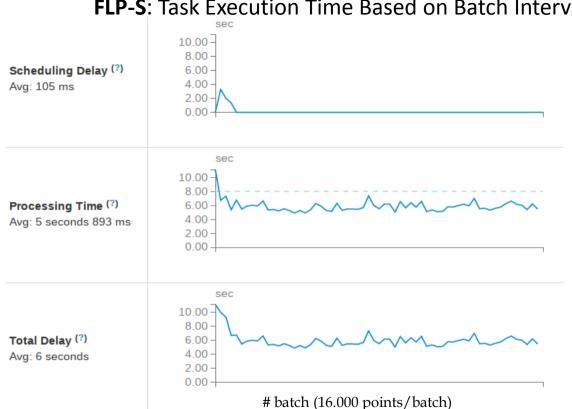


FLP-S: LEMD/LEBL (IFS radar)



FLP-S: Horizontal & vertical error for complete flights with sampling rate 4 seconds





FLP-S: Task Execution Time Based on Batch Interval

FLP-S: Performance metrics for 18 ·10⁶ points, 4000 points/sec processing in 8 secs



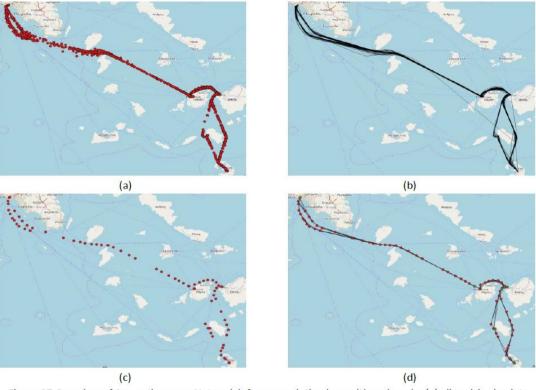
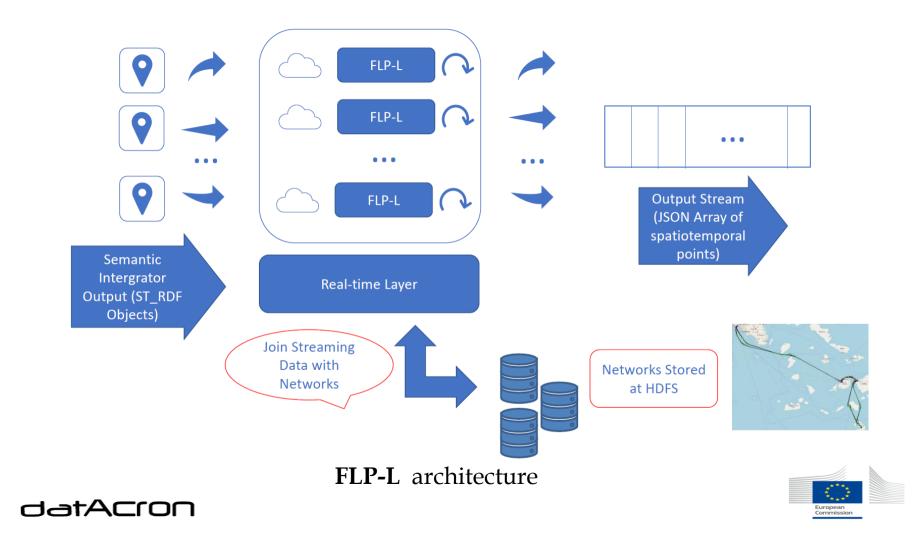
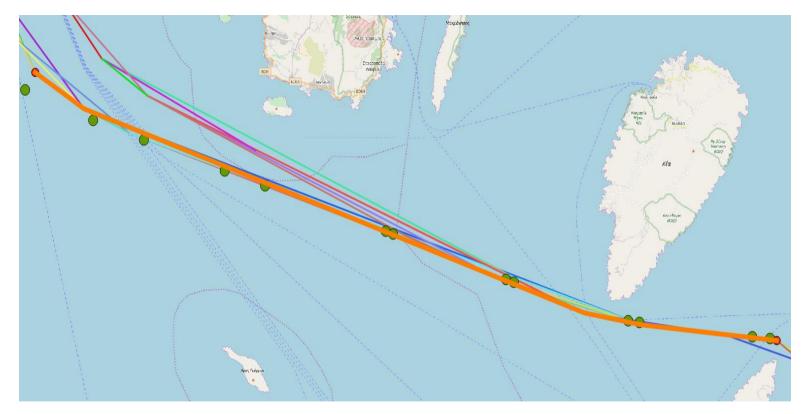


Figure 27 Overview of Semantic-aware Network inference solution in maritime domain: (a) all enriched points from input, (b) semantic trajectories formed from enriched points (c) semantic nodes extraction (d) semantic path discovery.

FLP-L: Routes discovery & network construction



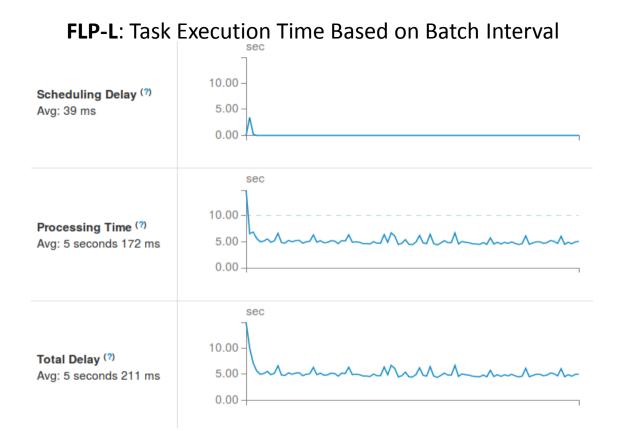




FLP-L: Red left = knn-based "start", Red right = knn-based "end", orange line = matched path



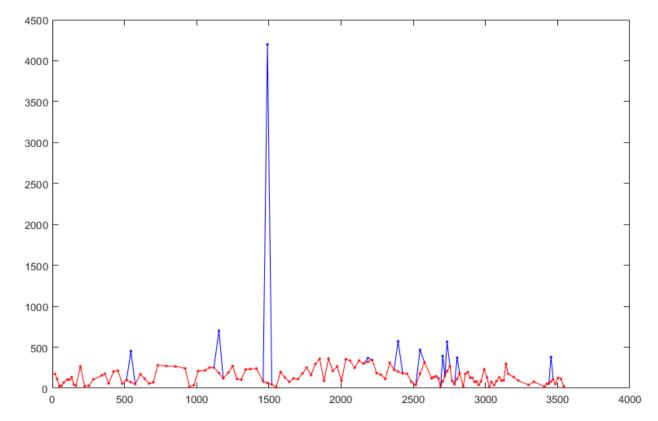




FLP-L: Performance metrics for 25 ·10⁶ points, 6000 points/sec processing in 10 secs

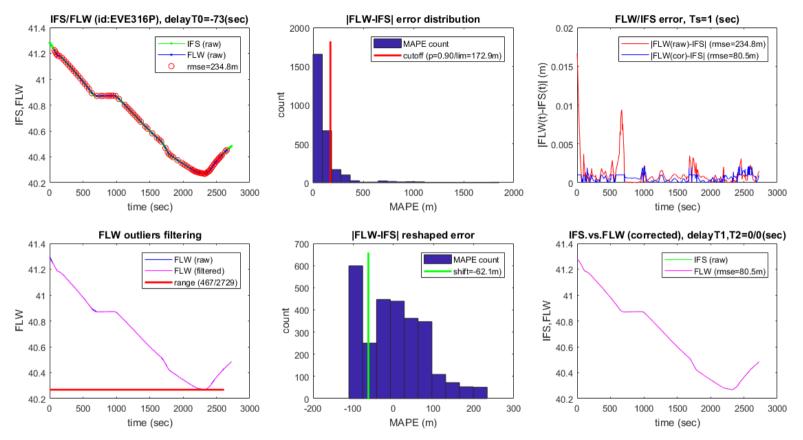
HatA





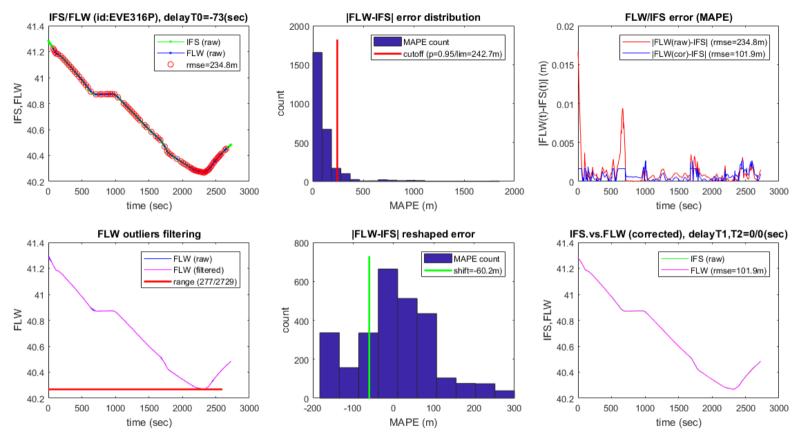
Cross-streaming: IFS/FLW deviations example (blue=orig./red=filtered)

European Commission



Cross-streaming: IFS/FLW statistical filter design ("strict", *p*=0.90)

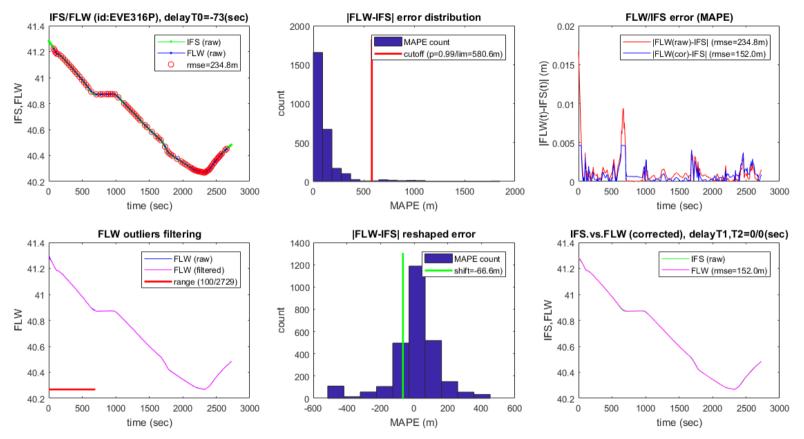




Cross-streaming: IFS/FLW statistical filter design ("medium", *p*=0.95)

European Commission





Cross-streaming: IFS/FLW statistical filter design ("relaxed", *p*=0.99)

European









This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Grant Agreement No 780754.

Track&Know

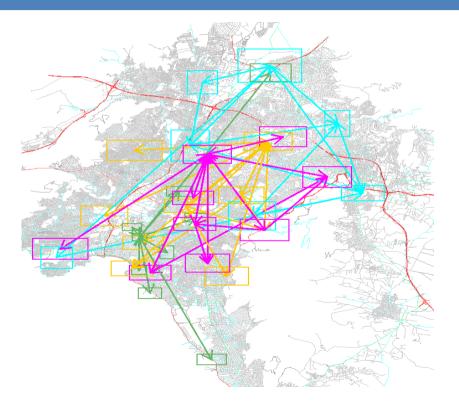




Track & Know

- Funded by the European Union's Horizon 2020
- Consortium of 14 partners from 9 different countries
- Research, develop and exploit a new software framework to increase efficiency of Big Data
- Applications in transport, mobility, motor insurance and health sectors
- Develop user-friendly toolboxes that will be readily applicable
- Validation in real-world pilots





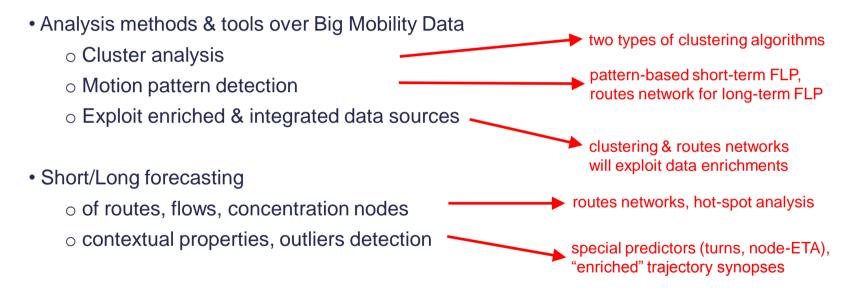
Big Data KPIs

Performance KPI	Target Value
Time-to-realization reduction	40% - 60%
Query time reduction	>15%
Data Load balancing time reduction for distributed query execution	>5%
Data Load balancing volume reduction for distributed query execution	>25%
Unstructured Data processing time reduction	>10%
Structured Data processing throughput improvement	>20%
Big Mobility Data pattern detection improvement	>30%
Big Mobility Data forecasting accuracy improvement	>40%
Complex Event Recognition processing improvement	>8%
Visualisation processing time reduction for interactive Mobility Data	>15%
Visualisation processing time reduction for aggregated Mobility Data	>10%



UPRC – WP4 planned work (M10+)

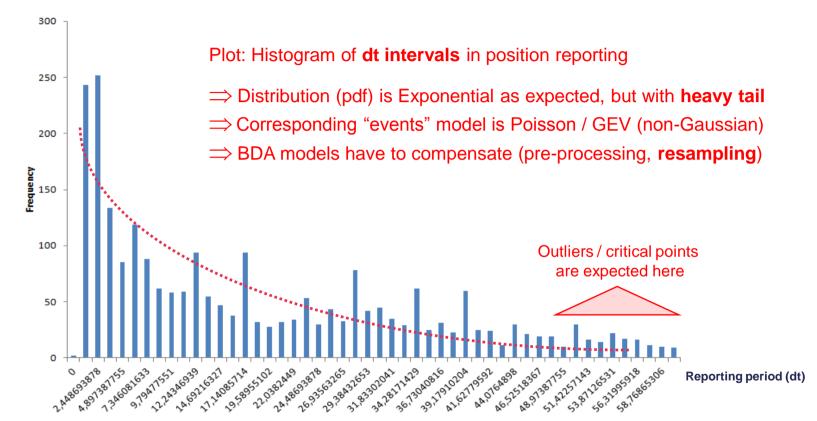
T4.1: "Analytics for mobility patterns detection for forecasting"



 \Rightarrow Also, from toy scenarios: <u>driving profiles</u>, fuel consumption, etc

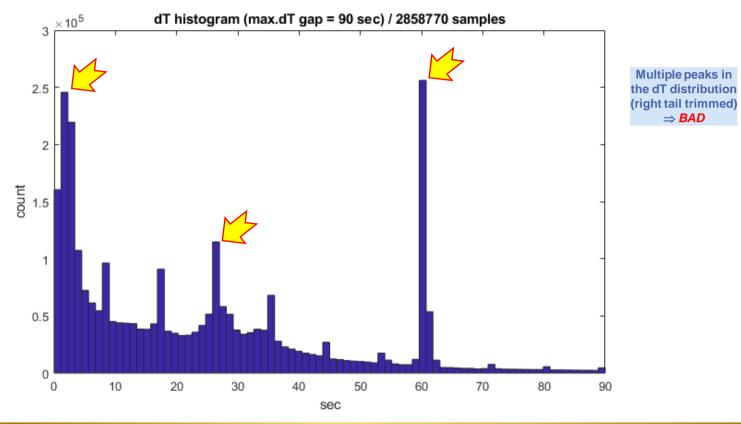


UPRC – WP4 status update (M11)



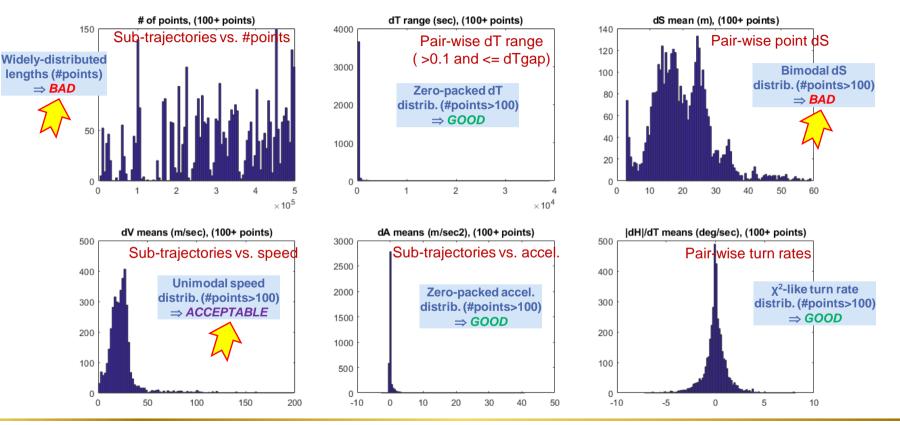


UPRC – WP4 status update (M11)





UPRC – WP4 status update (M11)









EU- H2020 ICT/ BigDataStack:

High-performance Data-centric Stack for Big Data Applications and Operations [www.bigdatastack.eu/], 2018-20

• EU-H2020 ICT/Track and Know:

Big Data for Mobility Tracking Knowledge Extraction in Urban Areas [trackandknowproject.eu/], 2018-20

- EU- H2020 ICT/ datAcron: Big Data Analytics for Time Critical Mobility Forecasting [datacron-project.eu/], 2016-18
- EU- H2020 SESAR/ DART: Data-driven Aircraft Trajectory Prediction Research [dart-research.eu], 2016-18
- GR/ RoadRunner: Scalable and Efficient Analytics for Big Data [platforms.gr/roadrunner/], 2014-15
- EU- FP7 Marie Curie/ **SEEK**: Semantic Enrichment of Trajectory Knowledge Discovery [<u>www.seek-project.eu</u>], 2012-15
- EU- FP7 ICT/ DATASIM: Data Science for Simulating the Era of Electric Vehicles [www.uhasselt.be/datasim], 2011-14
- EU- FP7 Marie Curie/ CloudIX: Cloud-based Indexing and Query Processing [research.idi.ntnu.no/cloudix/], 2011-13













Big (spatial-temporal) data management & query processing

- P. Tampakis et al. (2018) Distributed trajectory join processing using MapReduce. Submitted.
- C. Doulkeridis et al. (2017) Parallel and distributed processing of spatial preference queries using keywords. Proceedings of EDBT.
- F. Gryllakis et al. (2017) Searching for spatio-temporal-keyword patterns in semantic trajectories. Proceedings of IDA.
- M. Saouk et al. (2016) Efficient processing of top-k joins in MapReduce. Proceedings of Big Data.
- S. Sideridis et al. (2016) On querying and mining semantic-aware mobility timelines. Int. J. Data Science and Analytics, 2(1).
- D. Pertesis & C. Doulkeridis (2015) Efficient skyline query processing in SpatialHadoop. Information Systems, 54(C).
- C. Doulkeridis & K. Nørvåg (2014) A survey of large-scale analytical query processing in MapReduce. VLDB Journal, 23(3).

Big (spatial-temporal) data analytics & mining

- H. Georgiou et al. (2018) Predicting the next steps of moving objects: a survey. Under preparation.
- H. Georgiou et al. (2018) Semantic-aware aircraft trajectory prediction using flight plans. Submitted.
- G.A. Vouros et al. (2018) Big data analytics for time critical mobility forecasting: recent progress and research challenges. Proceedings of EDBT.
- N. Pelekis et al. (2017) On temporal-constrained sub-trajectory cluster analysis. Data Mining and Knowledge Discovery, 31(5).
- P. Nikitopoulos et al. (2016) BigCAB: Distributed hot spot analysis over big spatio-temporal data using Apache Spark (GIS Cup). Proceedings of ACM SIGSPATIAL - GIS.
- N. Pelekis et al. (2016) Simulating our LifeSteps by example. ACM Transactions on Spatial Algorithms and Systems, 2(3).





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"...Our goal is to address the challenging problems related to the wealth data, by advancing research and producing solutions to real world problems related to efficient and scalable management of Big Data, including gathering and cleansing data, storing and indexing data, analyzing and mining data."







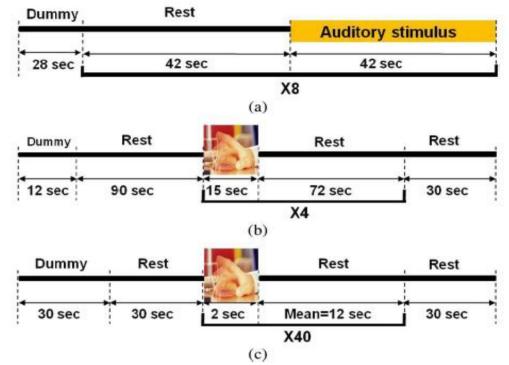
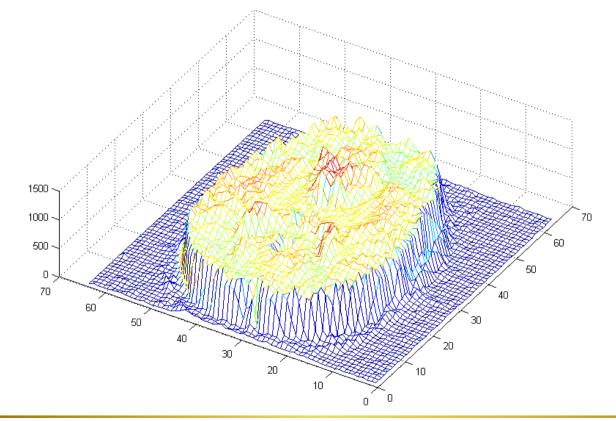
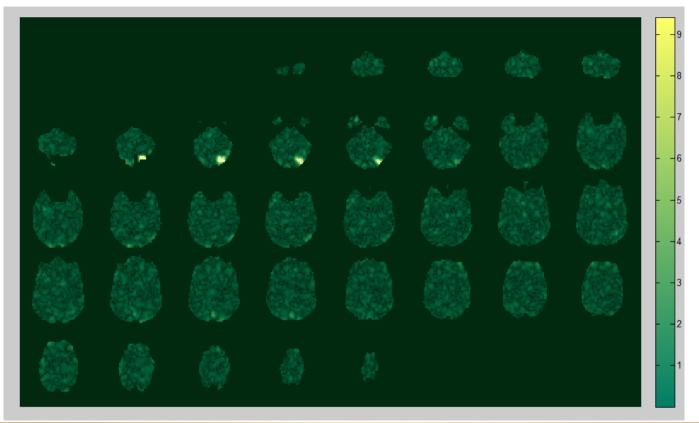


Fig. 3. Experimental paradigms for (a) auditory stimulus tasks, (b) block paradigm right finger tapping tasks, and (c) event-related right finger tapping tasks.

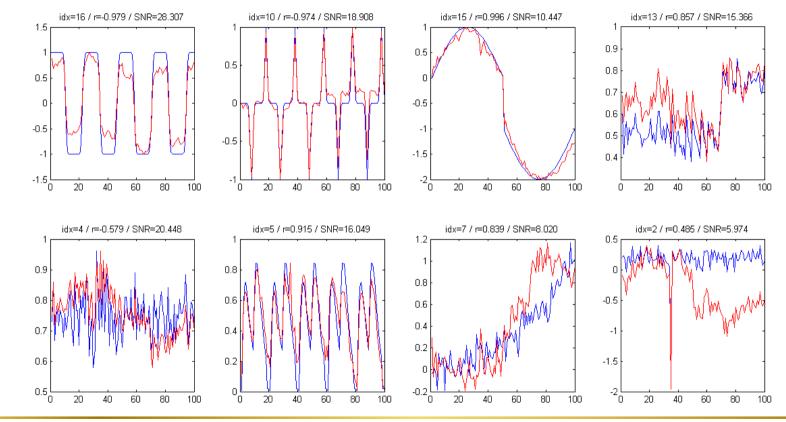




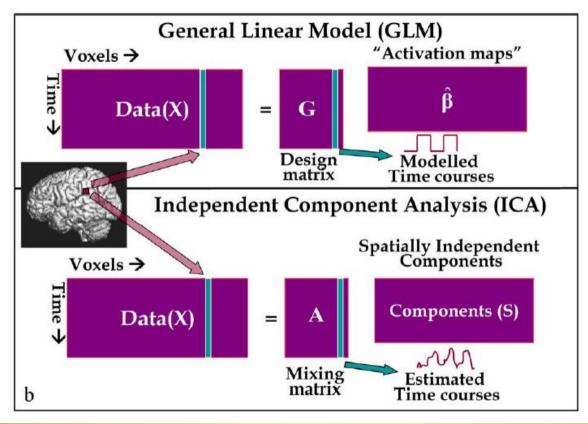




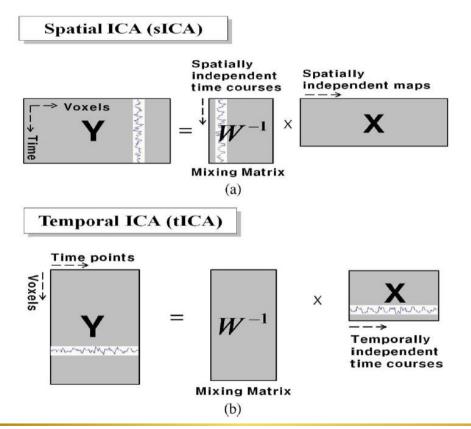




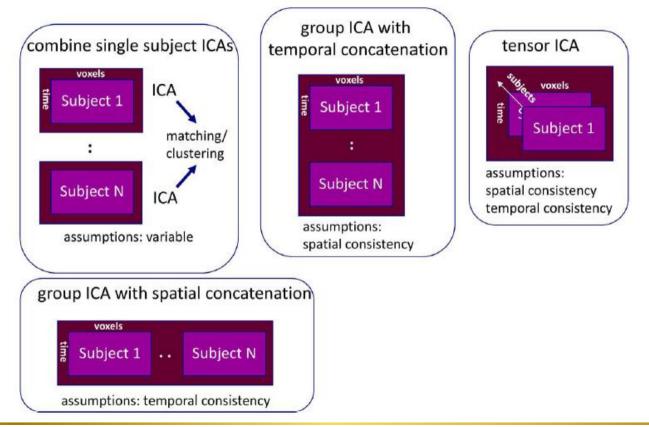














Dsize=12	Ca	Cm	Cam
fastICA	0.776 (0.917)	0.849 (0.971)	0.812 (0.944)
FIDL	0.833 (0.961)	0.801 (0.921)	0.817 (0.941)
sparse GLM	0.702 (0.764)	0.714 (0.800)	0.708 (0.782)
Proposed	0.846 (0.950)	0.853 (0.950)	0.850 (0.950)

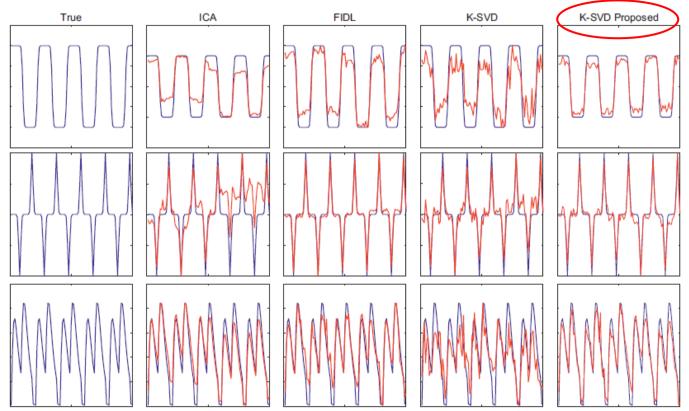
 Table 1. Performance with Dictionary size 12

Dsize=20	Ca	Cm	Cam
fastICA	0.776 (0.917)	0.849 (0.971)	0.812 (0.944)
FIDL	0.836 (0.960)	0.792 (0.895)	0.814 (0.927)
sparse GLM	0.798 (0.914)	0.697 (0.793)	0.747 (0.853)
Proposed	0.862 (0.972)	0.861 (0.952)	0.861 (0.962)

Table 2. Performance with Dictionary size 20

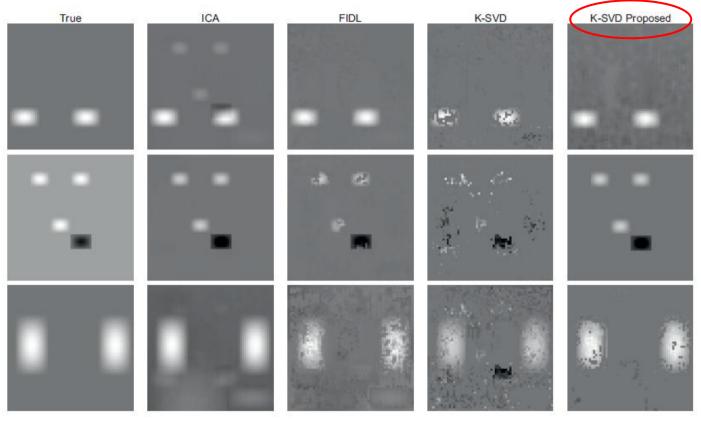
* See: Y. Kopsinis, H. Georgiou, S. Theodoridis (2014). "fMRI Unmixing Via Properly Adjusted Dictionary Learning", 22nd European Signal Processing Conference (EUSIPCO 2014), 1-5 Sept 2014 @ Lisbon, Portugal.





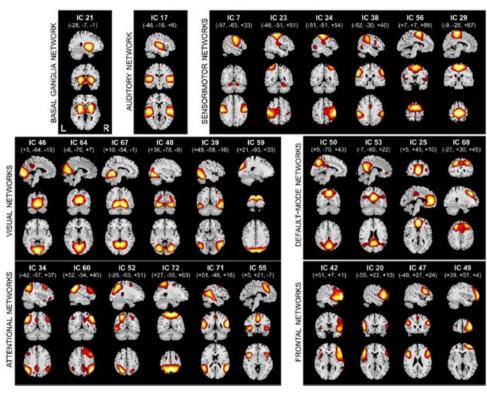
* See: Y. Kopsinis, H. Georgiou, S. Theodoridis (2014). "fMRI Unmixing Via Properly Adjusted Dictionary Learning", 22nd European Signal Processing Conference (EUSIPCO 2014), 1-5 Sept 2014 @ Lisbon, Portugal.



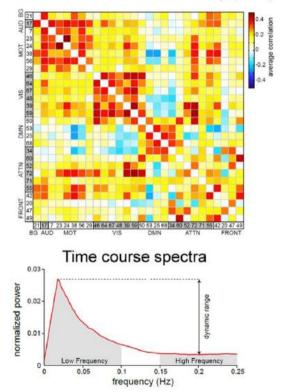


* See: Y. Kopsinis, H. Georgiou, S. Theodoridis (2014). "fMRI Unmixing Via Properly Adjusted Dictionary Learning", 22nd European Signal Processing Conference (EUSIPCO 2014), 1-5 Sept 2014 @ Lisbon, Portugal.

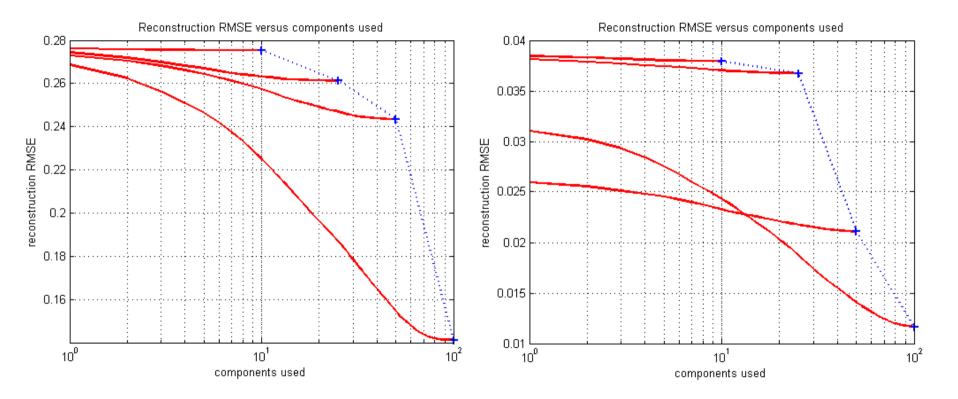
Component spatial maps



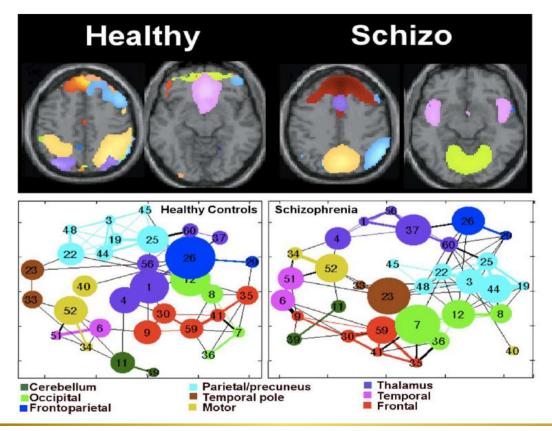
Functional network connectivity (FNC)







* See: H. Georgiou (2017). "Intrinsic dimension estimation of the fMRI space via sparsity-promoting matrix factorization", 21st Panhelenic Conference in Informatics (PCI 2017), 28-30 Sept 2017 @ Larisa, Greece (ACM).





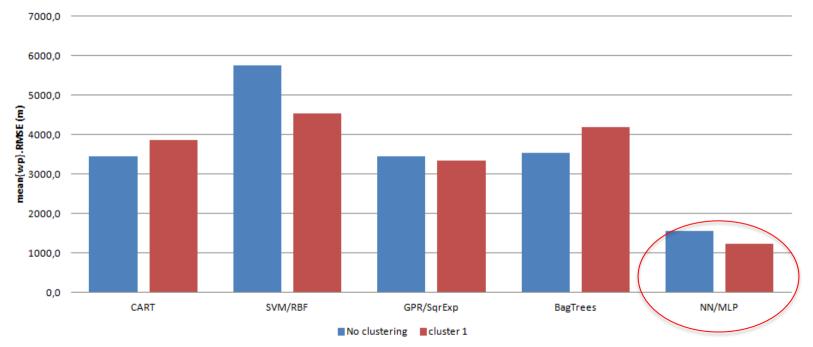
Questions

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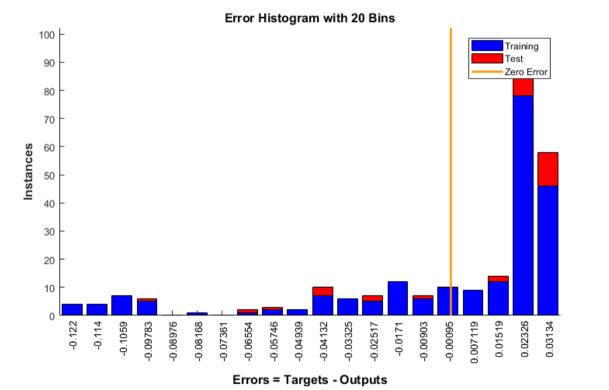


Generalization tests: Non-linear Regressors (cross-val: k=10...20) LEMD2LEBL / inp: {FP3d+AP} -> outp: {RT(j).Lat}



Non-linear Regressors (NN)

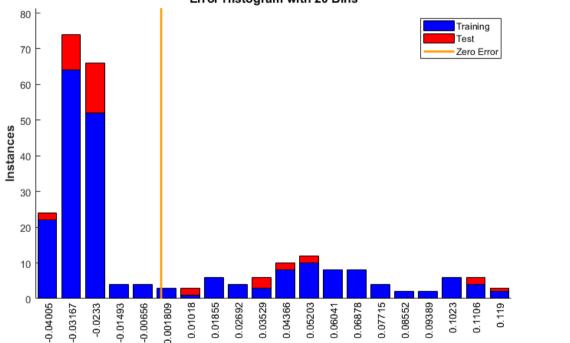




LEMD2LEBL / cluster 1 / wp8 (tst=15%): input=FP(3D)+AP \rightarrow output=Lat (NN)

Non-linear Regressors (NN)





Error Histogram with 20 Bins

Errors = Targets - Outputs

LEMD2LEBL / cluster 1 / wp10 (tst=15%): input=FP(3D)+AP \rightarrow output=Lat (NN)

Hybrid Clustering-Pred.Model (HMM, LR, DT, NN)



• Stage-2: HMMs vs. Linear Regressors:

- > LR improves accuracy (9-34%) vs. HMM across all configurations
- clustering/LR better (up to 16,5%) than clustering/HMM
- Non-clustered vs. Clustered dataset (HMM, LR, CART):
 - Clustering improves accuracy across all models & dimensionality (14-26,5%)
 - > Misaligned samples in clustering produces instability in stage-2 pred. models

^{*} See: H. Georgiou et al. (2018), Semantic-aware aircraft trajectory prediction using flight plans. (submitted)

Hybrid Clustering-Pred.Model (HMM, LR, DT, NN)



• LR (primary choice) vs. CART (as alternative):

➢ LR: (+) more resilient to "noise" in training, (-) less robust in generalization

- > CART: (-) less resilient to "noise" in training, (+) more robust in generalization
- > Complementary behavior, hints for combining them per-dimension
- Per-dimension comments, "active window" (LR), dataset properties:
 - > Lat. is inherently much more difficult to predict, possibly due to E/W orientation
 - Some LR use only "local neighbor" of ref. points (small coefficients elsewhere)

Hybrid Clustering-Pred.Model (HMM, LR, DT, NN)



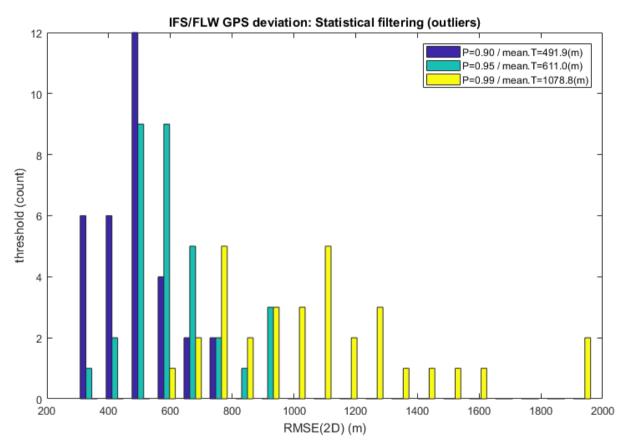
- Advanced non-linear regressors, generalization (vs. LR, CART):
 - > More resilient to "noise" in training, better generalization & stability
 - > Clustering (stage-1) becomes less important with more robust regressors
 - > NN: seems the best tradeoff between complexity vs. performance
- NN/MLP full experimental assessment:
 - > Lat. is still much more difficult to predict, possibly due to E/W orientation
 - > Multi-linear regression (3D output) seems much more difficult to achieve

WP2-Single Trajectory Prediction Issues, risks & contingencies:



- FP/RP "enriched" ref. points provide only a coarse trajectory as time series
- Many large deviations between "intended" (FP) and actual route (RT)
- Clustering is always beneficial in linear (HMM, LR) and multi-linear (CART)
- > ...but becomes less important when robust non-linear regressors are used
- > Error analysis (pred) shows few large peaks ("heavy" right tail in pdf)
- Dimensionality analysis: full 3-D FP input can be reduced (d=5...13 << 48)</p>
- ✓ Full-resolution (raw) IFS flight route may be combined with FP "constraints"
 ✓ Complexity vs. Performance tradeoff in regressors is a design-time decision

^{*} See: H. Georgiou et al. (2018), Semantic-aware aircraft trajectory prediction using flight plans. (submitted)



Cross-streaming: IFS/FLW outliers distribution vs. *p*-thresholds

datAcron



General Task

- Factor the data matrix as Y pprox AB aiming at getting A pprox T and B pprox S
- There is an infinite number of such factorizations
- A priori information information about the characteristics of B and/or ${\cal A}$ need to get imposed

fMRI unmixing approaches

• fMRI unmixing via Dictionary Learning: Factor $Y \approx AB$ under the constraint that the columns of B are sparse.

$$\min_{A,B} \|Y - AB\|_{\mathsf{F}}^2, \text{ s.t. } \|B_{\cdot,j}\|_0 \le K, \ j = 1 \cdots n,$$
(1)

- Fast and Incoherent Dictionary Learning (FIDL), [Abolghasemi 2013]:
 - **1** Incoherence is imposed in A,
 - elatively low complexity per iteration,
 - improved performance to both synthetic and real fMRI data.

* See: Y. Kopsinis, H. Georgiou, S. Theodoridis (2014). "fMRI Unmixing Via Properly Adjusted Dictionary Learning", 22nd European Signal Processing Conference (EUSIPCO 2014), 1-5 Sept 2014 @ Lisbon, Portugal.

Detect split atoms and merge them back

• Split maps corresponding to the same FBN are functionally associated, therefor should exhibit similar activation time-courses

• Spatial Maps Merging:

- Construct, m, the vector with indices of a set of highly correlated time courses.
- 2 Set, β the union of the supports of the spatial maps indexed in m.
- ${f 0}$ Solve for a and b the optimization problem

$$\min_{\boldsymbol{a},\boldsymbol{b}} \|\boldsymbol{A}_{\cdot,\boldsymbol{m}} \boldsymbol{B}_{\boldsymbol{m},\boldsymbol{\beta}} - \boldsymbol{a} \boldsymbol{b}^T \|_{\mathsf{F}}^2$$

Rank-1 best approximation (via truncated SVD).

\bigcirc Re-initialize the rest of the atoms indexed in m.

• Time Courses Merging: It can be done in exactly the same way.

• Both merging tasks need not to be performed in each K-SVD iteration.

