

# Knowledge Representation and Reasoning

Logic and Reasoning in  
Uncertain Conditions

# Reasoning in Uncertainty

- Probabilities and Uncertainty
- The Bayes rule
- Measures of Belief and Disbelief
- Certainty Factors (CF)
- Hypotheses and Evidences
- Logical operators and CF
- Inference rules and CF

# Probability Theory

- In real world, events happen with a certain **probability**, usually the result of many factors
- Some of these factors are independent with each other, i.e. happen independently
- Others usually happen (or not happen) together at the same time.
- Example: the event “rain” happens with a probability that depends on dependent factors “cloud” and “cold”

# The Bayes Theorem

- In order to calculate “joined” probabilities we use the Bayes rule:

$$P(H|E_k) = P(E_k|H)P(H) / \text{SUM}\{P(E_i|H)P(H)\}$$

$H$  = hypothesis (e.g.  $H$ =“rains”)

$E_k$  = evidence  $k$  (e.g.  $E_k$ =“cloud”)

$H|E_k$  = probability of “rain” when there is “cloud”

$E_k|H$  = probability of “cloud” when “rain” (past history)

# Certainty Factors

- Bayes rule becomes too complex when there are many evidences  $E_i$  that are dependent
- Instead, we often use approximate models for describing levels of certainty
- **Certainty Factors** (Shortliffe, 1976): first used in the expert system MYCIN
- They describe hypotheses and evidences in terms of “certainty” about their truth

# Measures of Belief and Disbelief

Measure of Belief (MB):

$$\begin{aligned} MB(H|E) &= 1 , \text{ if } P(H)=1 \\ &= \max\{P(H|E), P(H)\} / (1-P(H)) , \text{ if } P(H)<1 \end{aligned}$$

Measure of Disbelief (MD):

$$\begin{aligned} MD(H|E) &= 1 , \text{ if } P(H)=0 \\ &= (P(H) - \min\{P(H|E), P(H)\}) / P(H) , \text{ if } P(H)<1 \end{aligned}$$

# Certainty Factors

Certainty Factor (CF):

$$\mathbf{CF(H|E) = MB(H|E) - MD(H|E)}$$

$$0 \leq MB(H|E) \leq 1 , \quad 0 \leq MD(H|E) \leq 1$$

thus:  $-1 \leq CF(H|E) < +1$

## CF: Multiple evidences

What if new evidence  $E_k$  becomes available?

$$MB(H|E_1, E_2) = MB(H|E_1) + MB(H|E_2) * [1 - MB(H|E_1)]$$

$$MD(H|E_1, E_2) = MD(H|E_1) + MD(H|E_2) * [1 - MD(H|E_1)]$$

$$CF(H|E_1, E_2) = MB(H|E_1, E_2) - MD(H|E_1, E_2)$$

# CF: Uncertain evidences

What if evidence E is not 100% certain to be true ?

$MB(H|E)$  = normal MB value for H based on E

- If E comes from “measurements” M then:  
 $CF(E|M)$  = certainty of E based on M
- The updated value for  $MB(H|E)$  now becomes:  
 $MB(H|E)^{\text{new}} = MB(H|E) * \max\{ 0, CF(E|M) \}$

## CF: Composite hypotheses

What if composite hypotheses are to be evaluated?

$$\text{MB}(\text{H1 AND H2 } | \text{E}) = \min\{ \text{MB}(\text{H1} | \text{E}), \text{MB}(\text{H2} | \text{E}) \}$$

(similarly for MD)

$$\text{MB}(\text{H1 OR H2 } | \text{E}) = \max\{ \text{MB}(\text{H1} | \text{E}), \text{MB}(\text{H2} | \text{E}) \}$$

(similarly for MD)

$$\text{MB}(\text{NOT H} | \text{E}) = 1 - \text{MB}(\text{H} | \text{E})$$

(similarly for MD)

## CF: IF-THEN inference rules

How do we calculate CF for logical IF-THEN rules?

*rule(R): IF “E” THEN “H” with certainty CF(R)*

- Case:  $CF(H) > 0, CF(R) > 0$  then:

$$CF(H|E) = CF(H) + CF(R) * (1 - CF(H))$$

- Case:  $CF(H) < 0, CF(R) < 0$  then:

$$CF(H|E) = CF(H) + CF(R) * (1 - CF(H))$$

- Case:  $CF(H) * CF(R) < 0$  (i.e. different signs) then:

$$CF(H|E) = (CF(H) + CF(R)) / (1 - \min\{|CF(H)|, |CF(R)|\})$$

## P.C. – Readings

- S. J. Russell, P. Norvig, “Artificial Intelligence: A Modern Approach”, 2nd/Ed, Prentice Hall, 2002.  
[see: ch.19]